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NEW GENERALIZED XLINDLEY DISTRIBUTION: ITS PROPERTIES AND APPLICATION TO LIFETIME DATA

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Abstract: **This paper proposes a new distribution called New Generalized XLindley distribution (NGXLD), this distribution is generated as an extension of Xlindley distribution and hence the name proposed. Statistical properties like the maximum likelihood estimation method and moment were used in estimating the parameters of the proposed distribution. The Kolmogorov Smirnov goodness of fit was tested with four retrieved different datasets, compared with some selected existing model and shows more efficient than the considered models.**

Keywords: **Coronavirus, Harmonic Mean, Kolmogorov Smirnov, Maximum Likelihood Estimation, Quasi XLindley Distribution, Xlindley distribution, Weighted Distribution.**

I. INTRODUCTION

Researchers in distribution theory have focused on developing extended families of continuous distributions to enhance the modeling flexibility of traditional probability distributions. A new type of probability distribution arose due to the widespread availability of additional components[1]. The accuracy and sufficiency of data gathered from natural occurrences as well as the precision with which the distribution tail shape is described enhance when a specific parameter is added to a known probability function. We resorted to probability to lower the risk element in many sectors and productions to minimise cost and time since these events are crucial and are surrounded by complexity and hazard. In the statistical literature, many lifespan distributions have been established to provide data modelling in these applied disciplines more flexibility [2]. Lindley Distribution was introduced by Lindley [3], Lindley distribution has monotonically decreasing hazard rate has its advantage over exponential distribution [4]. XLindley distribution [5] Quasi XLindley Distribution [6], Power XLindley Distribution [7], Inverse XLindley distribution [2]. The New Generalized Xlindley Distribution (NGXLD), is an extension of the Quasi Xlindley distribution. The NGXLD is developed by incorporating weighting mechanisms into the Quasi Xlindley model, thereby broadening its applicability in various statistical analyses.

The pdf of Lindley distribution of random variable X, with scale parameter γ is given by:

$$f(x;\gamma) = \frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}; \qquad x > 0, \gamma > 0 \tag{1}$$

The idea of this work is to extend Quasi Xlindley Distribution called a New Generalized XLindley Distribution (NGXLD) with the hope that it will attract many applications in different disciplines. On applying the weighted version, the third parameter indexed, to this distribution, it is expected to be more flexible to describe different lifetime data than its sub-models.

In this paper, a New Generalized Quasi XLindley distribution which includes Lindley distribution, Exponential Lindley distribution and Quasi XLindley distribution as particular cases, has been proposed and discussed. The NGXLD is developed by incorporating weighting mechanisms into the Quasi Xlindley model, thereby broadening its applicability. Weighted distributions are utilized to modulate the probabilities of the events as observed and transcribed. [8, 9]. The hazard

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rate function and Survival function of the distribution have been derived and their shapes have been discussed for varying values of the parameters. The estimation of its parameters has been discussed using maximum likelihood method. Finally, the goodness of fit and the applications of the distribution have been explained through four lifetime data and the fit has been compared with the Quasi XLindley distribution, Power Xlindley distribution and Inverse Xlindley distribution.

II. METHODS

New Generalized XLindley Distribution

In this section, we proposed a new generalized of Xlindley distribution through the incorporation weighted mechanism into the Quasi Xlindley distribution with scale parameter ϕ, γ and δ.

The method of moment, the coefficient of variation, Coefficient of Skewness, Coefficient of Kurtosis, Index of Dispersion, the survival function, the hazard function, and maximum likelihood estimation were considered by the study. The pdf of New Generalized XLindley distribution for a random variable X with scale parameter ϕ and shape parameters δ and γ is defined as

$$f(x;\phi,\gamma,\delta)=\frac{\gamma^{(1+\delta)}}{1+\phi+\delta}\frac{x^{\delta}(\phi+\gamma+x\gamma+\phi\gamma)}{\Gamma(1+\delta)}e^{-x\gamma};x>0,\phi,\gamma,\delta>0 \quad (2)$$

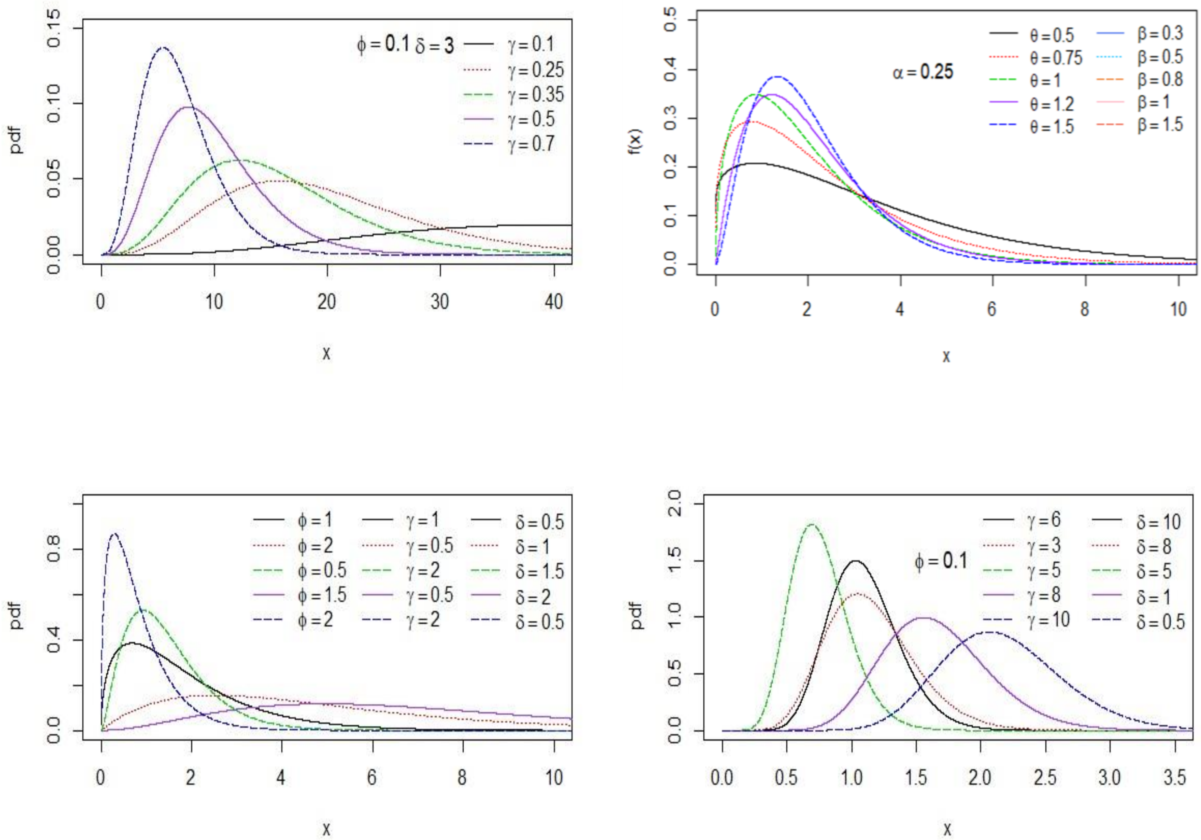

Figure 1: The pdf of NGXLD for various values of parameters

The graphs presented in Figure 1 shows the pattern of the pdf of the proposed NGXLD at various values of ϕ, γ and δ. The pdf shows different shapes which makes it flexible to capture different shapes as may be exhibited by different data fields.

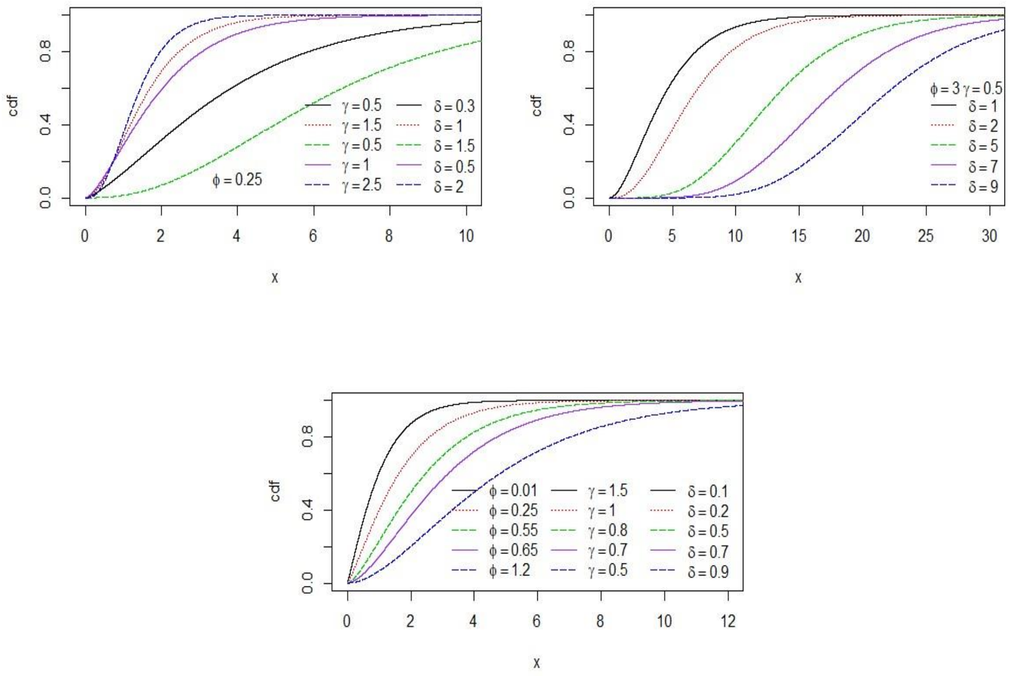

Figure 2: The cdf of NGXLD for various values of parameters

Moment

The moments of distribution are used to describe the characteristics or shape of the distribution. The rth moment of a Continuous random variable X:

$$E(X^r) = \mu_r' = \int_0^\infty x^r f(x)dx \qquad (3)$$

Where ‘r’ is a positive integer.

Hence, the r^{th} moment of a random variable X with the NGXLD is obtained as:

$$E(X^r) = \mu_r' = \frac{\gamma^{-r}(1+r+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+r+\delta)}{1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta)} \qquad (4)$$

The first four moments of NGXLD are:

$$\mu_1' = \frac{(1+\delta)(2+\phi+\delta+\gamma+\phi\gamma)}{\gamma(1+\phi+\delta+\gamma+\phi\gamma)} \qquad (5)$$

$$\mu_2' = \frac{(1+\delta)(2+\delta)(3+\phi+\delta+\gamma+\phi\gamma)}{\gamma^2(1+\phi+\delta+\gamma+\phi\gamma)} \qquad (6)$$

$$\mu_3' = \frac{(4+\phi+\delta+\gamma+\phi\gamma)\Gamma(4+\delta)}{\gamma^3(1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta)} \qquad (7)$$

$$\mu_4' = \frac{(5+\phi+\delta+\gamma+\phi\gamma)\Gamma(5+\delta)}{\gamma^4(1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta)} \qquad (8)$$

Given the random variable X having the NGXLD with parameters ϕ, γ and δ the variance, coefficient of variation, Skewness, Kurtosis, Index of Variation and Harmonic Mean is given by

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$$\sigma^2 = \frac{(1+\delta)(2+\delta^2+4\gamma+\gamma^2+\phi^2(1+\gamma)^2+2\phi(1+\gamma)(2+\delta+\gamma)+\delta(3+2\gamma)}{\gamma^2(1+\phi+\delta+\gamma+\phi\gamma)^2} \quad (9)$$

$$CV = \frac{\sqrt{2+\delta^2+4\gamma+\gamma^2+\phi^2(1+\gamma)^2+2\phi(1+\gamma)(2+\delta+\gamma)+\delta(3+2\gamma)}}{\gamma(1+\phi+\delta+\gamma+\phi\gamma)(2+\phi+\delta+\gamma+\phi\gamma)} \quad (10)$$

$$S_k = \frac{(4+\phi+\delta+\gamma+\phi\gamma)\Gamma(4+\delta)}{\gamma^3(1+\phi+\delta+\gamma+\phi\gamma)\left[\frac{(1+\delta)(2+\delta^2+4\gamma+\gamma^2+\phi^2(1+\gamma)^2+2\phi(1+\gamma)(2+\delta+\gamma)+\delta)(3+2\gamma)}{\gamma^2(1+\phi+\delta+\gamma+\phi\gamma)^2}\right]^{\frac{3}{2}}\Gamma(1+\delta)} \quad (11)$$

$$K_s = \frac{(1+\phi+\delta+\gamma+\phi\gamma)^3(5+\phi+\delta+\gamma+\phi\gamma)\Gamma(5+\delta)}{(1+\delta)^2(2+\delta^2+4\gamma+\gamma^2+\phi^2(1+\gamma)^2+2\phi(1+\gamma))(2+\delta+\gamma)+\delta(3+2\gamma))^2\Gamma(1+\delta)} \quad (12)$$

$$ID_s = \frac{(2+\delta^2+4\gamma+\gamma^2+\phi^2(1+\gamma)(2+\delta+\gamma)+\delta(3+2\gamma))}{\gamma(1+\phi+\delta+\gamma+\phi\gamma)(2+\phi+\delta+\gamma+\phi\gamma)} \quad (13)$$

$$HM = \left(\frac{\gamma}{1+\phi+\delta}\left[\frac{\phi+\gamma}{\Gamma(1+\delta)}+\frac{\delta(\gamma+\phi\gamma)}{\Gamma(1+\delta)}\right]\right)^{-1}. \quad (14)$$

Table 1: Statistical Properties of NGXLD for $\phi = 0.3$ and 1.0

Parameters			Statistic					
ϕ	γ	δ	μ	CV	Sk	Ks	ID	HM
0.3	0.3	0.3	7.1859	6.7935	6.0160	11.1203	2.9610	1.3660
		1.0	7.0508	54.3754	0.2665	0.2270	2.9969	0.4049
		2.5	6.9133	190.2733	0.1059	0.0385	3.0659	0.3442
		5.0	6.8211	525.4702	0.6429	0.0169	3.1388	3.7939
	1.0	0.3	1.7931	1.8368	1.1559	1.5262	0.8488	0.6648
		1.0	1.8333	5.9722	0.2569	0.2639	0.8582	0.2331
		2.5	1.8824	18.3832	0.2061	0.0832	0.8791	0.2393
		5.0	1.9211	49.3622	0.8062	0.0433	0.9054	3.0083
	2.5	0.3	0.5897	0.4315	0.6500	0.8139	0.3163	0.3166
		1.0	0.6162	1.1078	0.2551	0.2712	0.3206	0.1221
		2.5	0.6553	3.1808	0.2890	0.1183	0.3293	0.1447
		5.0	0.6932	8.3857	0.8070	0.0712	0.3409	2.0837
	5.0	0.3	0.2568	0.1245	0.5242	0.6542	0.1487	0.1690
		1.0	0.2682	0.2990	0.2575	0.2717	0.1507	0.0681
		2.5	0.2874	0.8410	0.3338	0.1361	0.1546	0.0872
		5.0	0.3094	2.2143	0.7678	0.0892	0.1602	1.3779
1.0	0.3	0.3	5.9770	20.4094	1.1559	1.5262	2.8293	0.3971
		1.0	6.1111	66.3580	0.2569	0.2639	2.8607	0.1923
		2.5	6.2745	204.2580	0.2061	0.0832	2.9302	0.2246
		5.0	6.4035	548.4688	0.8062	0.0433	3.0178	2.9412
	1.0	0.3	1.5349	2.0248	1.2531	2.3269	0.9181	0.2500
		1.0	1.6000	5.8400	0.4393	0.6255	0.9167	0.1250
		2.5	1.6923	17.7322	0.4499	0.2400	0.9200	0.1602
		5.0	1.7778	48.1728	0.9424	0.1303	0.9300	2.3077
	2.5	0.3	0.5260	0.3328	1.4171	3.2276	0.3866	0.1394
		1.0	0.5500	0.8975	0.6445	1.0929	0.3852	0.0714
		2.5	0.5895	2.7030	0.6865	0.4708	0.3837	0.0992
		5.0	0.6333	7.4389	1.0275	0.2690	0.3834	1.5789

	5.0	0.3	0.2374	0.0827	1.5578	3.9540	0.1974	0.0802
		1.0	0.2462	0.2163	0.8012	1.4945	0.1969	0.0417
		2.5	0.2621	0.6482	0.8522	0.6926	0.1963	0.0607
		5.0	0.2824	1.7979	1.0932	0.4158	0.1957	1.0345

Table 2: Statistical Properties of NGXLD for ϕ = 2.5 and 0.5

Parameters			Statistic					
ϕ	γ	δ	μ	CV	Sk	Ks	ID	HM
2.5	0.3	0.3	4.9141	29.9663	0.6500	0.8139	2.6362	0.0998
		1.0	5.1351	76.9337	0.2551	0.2712	2.6719	0.0697
		2.5	5.4610	220.8868	0.2890	0.1183	2.7440	0.1141
		5.0	5.7766	582.3375	0.8070	0.0712	2.8409	1.8824
	1.0	0.3	1.3151	2.0798	1.4171	3.2276	0.9666	0.0768
		1.0	1.3750	5.6094	0.6445	1.0929	0.9630	0.0519
		2.5	1.4737	16.8940	0.6865	0.4708	0.9592	0.0858
		5.0	1.5833	46.4931	1.0275	0.2690	0.9586	1.4769
	2.5	0.3	0.4733	0.2449	2.8674	10.4542	0.4736	0.0514
		1.0	0.4906	0.7012	1.3161	3.3016	0.4680	0.0336
		2.5	0.5220	2.2048	1.2094	1.3743	0.4582	0.0560
		5.0	0.5623	6.3064	1.3383	0.7604	0.4467	1.0105
	5.0	0.3	0.2216	0.0480	4.6498	23.1689	0.2724	0.0331
		1.0	0.2273	0.1447	2.0643	6.8022	0.2691	0.0212
		2.5	0.2383	0.4660	1.7292	2.8052	0.2629	0.0354
		5.0	0.2538	1.3571	1.6851	1.5638	0.2547	0.6621
5.0	0.3	0.3	4.2798	34.5925	0.5242	0.6542	2.4782	0.0297
		1.0	4.4697	83.0521	0.2575	0.2717	2.5116	0.0253
		2.5	4.7896	233.6127	0.3338	0.1361	2.5773	0.0528
		5.0	5.1562	615.0798	0.7678	0.0892	2.6692	1.0714
	1.0	0.3	1.1870	2.0672	1.5578	3.9540	0.9870	0.0255
		1.0	1.2308	5.4083	0.8012	1.4945	0.9847	0.0208
		2.5	1.3103	16.2054	0.8522	0.6926	0.9813	0.0422
		5.0	1.4118	44.9481	1.0932	0.4158	0.9784	0.8571
	2.5	0.3	0.4432	0.1921	4.6498	23.1689	0.5447	0.0196
		1.0	0.4545	0.5788	2.0643	6.8022	0.5381	0.0152
		2.5	0.4766	1.8639	1.7292	2.8052	0.5258	0.0295
		5.0	0.5077	5.4284	1.6851	1.5638	0.5093	0.6000
	5.0	0.3	0.2127	0.0282	11.7742	101.4873	0.3497	0.0141
		1.0	0.2162	0.1024	4.0557	20.9022	0.3447	0.0104
		2.5	0.2234	0.3510	2.8731	7.7816	0.3352	0.0197
		5.0	0.2341	1.0515	2.4797	4.2076	0.3218	0.4000

From table 1 and 2, the follows can be deduced

- at initial value of ϕ = 0.3 and δ -> increase leads to a reduction in the mean and an increase in variance, indicating that the distribution becomes more spread out with smaller mean values. Skewness and kurtosis decrease, suggesting the distribution becomes less asymmetric and less heavy-tailed.
- as δ -> increases, harmonic mean also decreases.
- as ϕ increases, the distribution becomes more concentrated with higher values of δ.
- larger γ values tend to increase kurtosis and skewness,
- δ modifies the central tendency and tail behavior of the distribution

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In general, ϕ determines the overall scale of the distribution, with higher values compressing it and reducing both the mean and skewness. The parameter γ influences the dispersion,

Hazard Function

The hazard function $h(x)$ of an event is the probability of the failure of the event at time x. It is the probability that an event will fail at a given time x. The hazard function $h(x)$ is given as:

$$h(x) = \frac{f(x)}{1 - F(x)} = \frac{f(x)}{s(x)} \tag{15}$$

the hazard function of NGXLD is given as

$$h(x, \phi, \gamma, \delta) = \frac{e^{-x\gamma}\gamma\left(\phi + \frac{(1+x)\gamma}{1+\gamma}\right)}{(1+\phi)\left(1 + \frac{x^{\delta}\gamma^{\delta}(x\gamma)^{-\delta}(-(1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta)+(\phi+\gamma+\phi\gamma)\Gamma(1+\delta,x\gamma)+\Gamma(2+\delta,x\gamma))}{(1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta)}\right)} \tag{16}$$

Figure 3 presented the shapes of the hazard function of the proposed NGXLD at selected values of ϕ, γ and δ. The plots show an increasing pattern which suggests that items are more likely to fail at the beginning after which they maintain a constant failure rate. This means that the failure may likely happen during the useful life of the item and failure occurs at random.

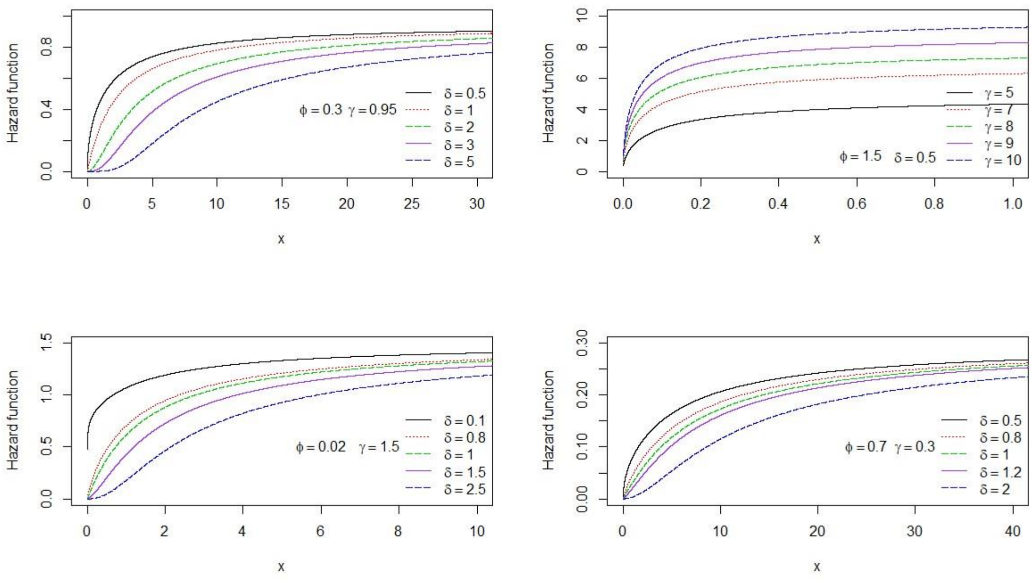

Figure 3: The hazard function of NGXLD for various choices of parameter

Survival Function

The survival function is a function that gives the probability that a patient, device, or other object of interest will survive past certain time [10]. Let the lifetime x be a continuous random variable with cumulative density function $F(x)$ and probability density function $f(x)$ on the interval $[0, \infty]$. survival function of NGXLD is given as: $S(x, \phi, \gamma, \delta) = 1 - F(x; \phi, \gamma, \delta)$

$$S(x, \phi, \gamma, \delta) = 1 + \frac{x^{\delta}\gamma^{\delta}(x\gamma)^{-\delta}\left(-(1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta) + (\phi+\gamma+\phi\gamma)\Gamma(1+\delta, x\gamma) + \Gamma(2+\delta, x\gamma)\right)}{(1+\phi+\delta+\gamma+\phi\gamma)\Gamma(1+\delta)}$$

Figure 4 presented the shapes of the survival function of the proposed NGXLD at selected values of ϕ, γ and δ.

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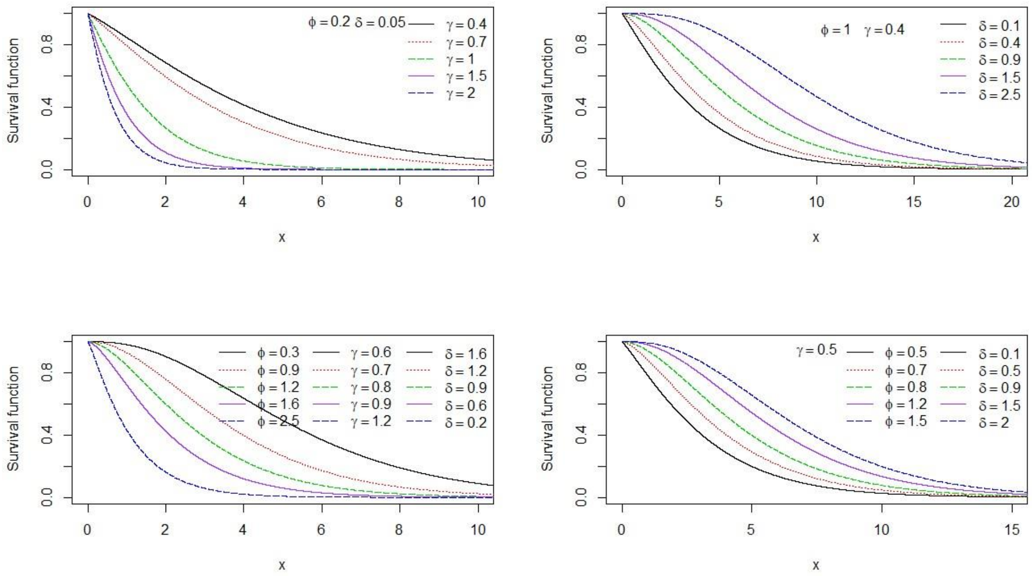

Figure 4: The survival function of NGXLD for various choices of parameter

Maximum Likelihood Estimation (MLEs)

The Maximum Likelihood Estimate (MLE) is a widely used method for estimating the parameters of an assumed probability distribution. This is because of MLE estimators have desirable properties such as consistency, asymptotic efficiency, and invariance. To obtain the maximum likelihood estimators of the parameters of the NGXLD, let $x_1,x_2,...,x_n$ be a random sample of size n from the NGXLD with the log-likelihood function is defined as

$$L(\phi,\gamma,\delta;x)=\prod_{i=1}^{n} f(x;\phi,\gamma,\delta,)$$

To find the MLE $\hat{\phi},\ \hat{\gamma},\ \hat{\delta}$ take the partial derivatives of the log-likelihood function with respect to ϕ, γ and δ, and set them to zero as: to differentiate with respect to γ, we ignore the terms without γ, we have

$$\ell(\phi,\gamma,\delta,x_i)=(1+\delta)ln(\gamma)+g(\phi,\gamma,\delta,x_i)$$

$$\frac{\partial\ell}{\partial\gamma}=\sum_{i=1}^{n}\left(\frac{1+\delta}{\gamma}+\frac{1+x_i+\phi}{\phi+\gamma+x_i\gamma+\phi\gamma}-n\right)$$

to differentiate with respect to ϕ, we ignore the terms without ϕ, we have

$$\ell(\phi,\gamma,\delta,x_i)=g(\phi,\gamma,\delta,x_i)-ln(1+\phi+\delta)$$

$$\frac{\partial\ell}{\partial\phi}=\frac{n}{1+\phi+\delta}+\sum_{i=1}^{n}\frac{1}{\phi+\gamma+x_i\gamma+\phi\gamma}$$

to differentiate with respect to δ, we ignore the terms without δ, we have:

$$\ell(\phi,\gamma,\delta,x_i)=(1+\delta)ln(\gamma)-ln(1+\phi+\delta)+\delta\sum_{i=1}^{n}ln(x_i)+g(\phi,\gamma,\delta,x_i)$$

$$\frac{\partial\ell}{\partial\delta}=n\ln(\gamma)-\psi(1+\delta)+\sum_{i=1}^{n}\ln(x_i)$$

These three natural log likelihood do not seem to be solved directly. However, the Fisher's scoring method was used to solve these equations and an Hessian Matrix of the log-likelihood function lnL, which consists of second-order partial derivatives of lnL with respect to the parameters ϕ,γ, δ. was obtained

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$$I(\phi,\gamma,\delta)=\begin{bmatrix}\frac{\partial^2 lnL}{\partial\phi^2} & \frac{\partial^2 lnL}{\partial\phi\partial\gamma} & \frac{\partial^2 lnL}{\partial\phi\partial\delta}\\ \frac{\partial^2 lnL}{\partial\gamma\partial\phi} & \frac{\partial^2 lnL}{\partial\gamma^2} & \frac{\partial^2 lnL}{\partial\gamma\partial\delta}\\ \frac{\partial^2 lnL}{\partial\delta\partial\phi} & \frac{\partial^2 lnL}{\partial\delta\partial\gamma} & \frac{\partial^2 lnL}{\partial\delta^2}\end{bmatrix}$$

where ϕ_o, γ_o and δ_0 are the initial values of ϕ, γ and δ respectively. These equations were solved iteratively till sufficiently close values of $\hat{\phi}, \hat{\gamma}$ and $\hat{\delta}$ are obtained.

Simulation Study

In this section, we evaluate $\hat{\phi}_{MLE}, \hat{\gamma}_{MLE}$ and $\hat{\delta}_{MLE}$ through a brief simulation study. The simulation study of the NGXLD is carried out by choosing random samples, say n = 50, 100, ..., 1000. These samples are obtained using the inverse cdf. The simulation study is conducted for the combination values ϕ =, γ = and δ = respectively. The judgment about the performances of the $\hat{\phi}_{MLE}$, $\hat{\gamma}_{MLE}$ and $\hat{\delta}_{MLE}$ are made by considering two evaluation criteria. These criteria are given by

Mean Square Error (MSE)

MSE $(\hat{\phi}_{MLE}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\phi}_i-\phi)^2$

Bias

Bias $(\hat{\phi}_{MLE}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\phi}_i-\phi)$

The above evaluation criteria are also computed for $\hat{\gamma}_{MLE}$ and $\hat{\delta}_{MLE}$. The simulation study is performed using the optim()R-function with argument method = "L-BFGSB". The simulation results are presented numerically in Tables 3.

Table 3: The numerical illustration of the Simulation Study of the NGXLD for ϕ = 1.0 , γ = 2.0 and δ = 1.0

n	Parameter	MLE	Biases	RMSE
15	ϕ	4.916×10^8	4.916×10^8	4.749×10^9
	γ	5.898	3.898	4.839
	δ	0.9046	−0.0954	1.519
85	ϕ	1.329×10^8	1.329×10^8	1.232×10^9
	γ	4.626	2.626	2.838
	δ	0.3085	−0.6915	0.9598
120	ϕ	6.584×10^7	6.584×10^7	3.161×10^8
	γ	4.558	2.558	2.748
	δ	0.2593	−0.7407	0.9623
250	ϕ	1.073×10^7	1.073×10^7	5.367×10^7
	γ	4.251	2.251	2.342
	δ	0.09957	−0.9004	0.9774
400	ϕ	7.513×10^6	7.513×10^6	5.194×10^7
	γ	4.172	2.172	2.235
	δ	0.06309	−0.9369	0.9841
800	ϕ	8.147×10^5	8.147×10^5	1.380×10^7
	γ	4.080	2.080	2.094
	δ	0.00753	−0.9925	0.9971
1500	ϕ	0.002801	−0.9972	0.9973
	γ	4.0718	2.0718	2.0797
	δ	0.001252	−0.9987	0.9988

Interpretation

From the results of the simulation of NGXLD in Table 3, the followings were observed:

1. Larger sample sizes produce more accurate (unbiased) estimators
2. As *n* increases (i.e. as $n \to \infty$), the values of $\hat{\phi}_{MLE}$, $\hat{\gamma}_{MLE}$ and $\hat{\delta}_{MLE}$ become more consistent and closer to their true values

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3. As n increases (i.e. as $n \to \infty$), RMSE of $\hat{\phi}_{MLE}$, $\hat{\gamma}_{MLE}$ and $\hat{\delta}_{MLE}$ tends to 0, which is expected since larger samples reduce the variance of the estimator.

4. As n increases (i.e. as $n \to \infty$), Biases of $\hat{\phi}_{MLE}$, $\hat{\gamma}_{MLE}$ and $\hat{\delta}_{MLE}$ tends towards to 0, suggesting that the estimators are asymptotically unbiased.

The negative bias for δ suggests a tendency to underestimate its value, particularly at smaller sample sizes. While this bias diminishes with increasing sample size, it highlights the need for sufficient data or bias-correction techniques to ensure reliable parameter estimation and accurate model predictions.

Figure 4 presented the shapes of the survival function of the proposed NGXLD at selected values of ϕ, γ and δ. The plots also show a decreasing rate of survival at time x.

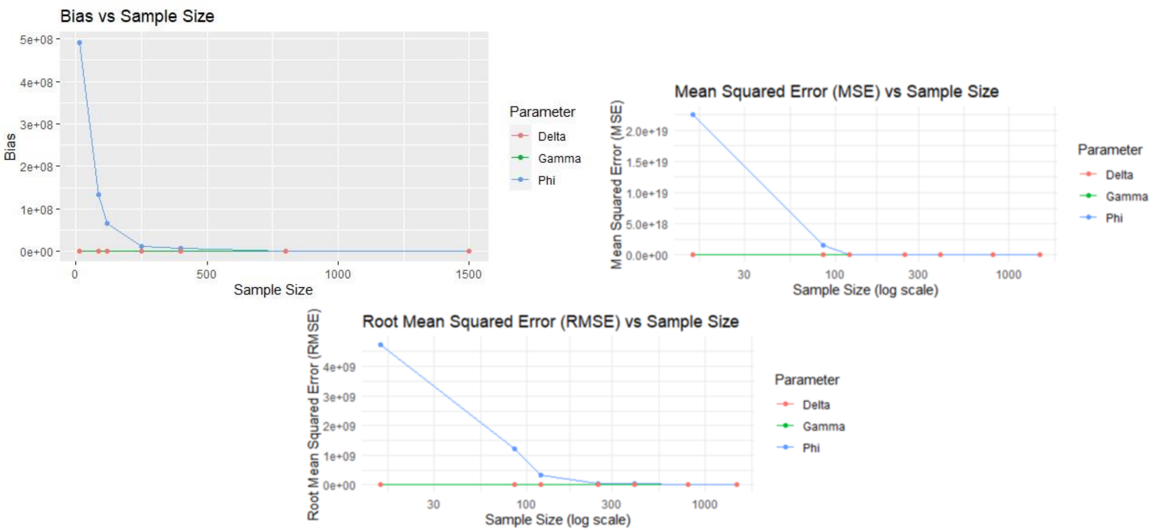

Figure 5: Simulation of NGXLD for different sample size

Models Selection Method

The model selection criteria considered in this thesis are the AIC (Akaike Information Criterion) by Akaike [11], AICC (Corrected Akaike Information Criterion) by Kletting and Glatting [12] HQIC (Hannan-Quinn Information Criterion) by Maïnassara and Kokonendji [13] and BIC (Bayesian Information Criterion) by Weakliem [14] and Kolmogorov Smirnov [15]. Where the AIC, AICC, HQIC, BIC and KS are obtained as follows:

$$AIC = 2k - 2ln(L) \tag{18}$$

$$AICC = AIC + \frac{k(k-1)}{n-k-1} \tag{19}$$

$$HQIC = -2\ln(L) + 6\ln(\ln(n)) \tag{20}$$

$$BIC = kln(n) - 2ln(L) \tag{21}$$

$$K.S = sup_x|F_n(x) - F_0(x)| \tag{22}$$

Where k is the number of parameters in the distribution and n is the number of observations.

III. APPLICATION

Four (4) real life datasets were used in this section to demonstrate the versatility of the proposed distribution.

Dataset 1: The First dataset represents the remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang [16]

Dataset 2: The second dataset represents 50 items failure times in weeks reported by Murthy et al . [17] quoted by Thashan et al. [18]

Dataset 3: The third dataset is related to the number of deaths due to corona virus in China from 23 January 2020, to 28 March 2020.

Dataset 4: This fourth dataset describes repair times for an airborne communication transceiver. It was given by Lemonte et al. [19]

Table 5: Parameter estimates and goodness of fit test statistics for datasets

Dataset	Model	MLE	-ll	AIC	AICC	HQIC	BIC	K-S	KS p-value
1	NGXLD	$\hat{\phi} = 0.2174$ $\hat{\gamma} = 0.1976$ $\hat{\delta} = 0.0000$	395.0971	796.1941	796.3876	804.7502	799.6705	0.1347	0.00000
	QXLD	$\hat{\phi} = 1267.4210$ $\hat{\gamma} = 0.1069$	414.3419	832.6838	832.7798	835.0014	838.3879	0.2691	0.3183
	PXLD	$\hat{\phi} = 0.5226$ $\hat{\gamma} = 0.5000$	573.9261	1151.8520	1151.9481	1157.5562	1154.1701	0.3188	0.0000
	IXLD	$\hat{\phi} = 6.8232$	536.0988	1074.1980	1074.229	1075.3560	1077.0500	0.7253	0.00000
2	NGXLD	$\hat{\phi} = 1.168201$ $\hat{\gamma} = 0.2324$ $\hat{\delta} = 0.0001$	133.5961	273.1923	273.7256	278.8677	275.3455	0.0774	0.0002
	QXLD	$\hat{\phi} = 487.8894$ $\hat{\gamma} = 0.1520$	144.2897	292.5793	292.8347	294.0356	296.4034	0.1329	0.3406
	PXLD	$\hat{\phi} = 0.6921$ $\hat{\gamma} = 0.5000$	175.01	354.02	354.2753	357.844	355.4762	0.2267	0.0097
	IXLD	$\hat{\phi} = 0.7605$	0.7605	661.3023	661.3856	662.0304	663.2143	0.6606	0.00000
3	NGXLD	$\hat{\phi} = 2.1020$ $\hat{\gamma} = 0.0289$ $\hat{\delta} = 0.2227$	253.6230	513.2461	513.7567	519.0415	515.4607	0.1003	0.0000
	QXLD	$\hat{\phi} = 2.0846$ $\hat{\gamma} = 0.0265$	323.6103	651.2206	651.411	652.951	655.5999	0.0659	0.6827
	PXLD	$\hat{\phi} = 10.0011$ $\hat{\gamma} = 0.4999$	906.5255	1817.0513	1817.241	1821.4322	1818.7812	1.0053	0.0000
	IXLD	$\hat{\phi} = 48.49743$	331.8675	665.735	665.7975	666.6002	667.9246	1.1038	0.2220
4	NGXLD	$\hat{\phi} = 0.1164$ $\hat{\gamma} = 0.4809$ $\hat{\delta} = 0.0000$	83.6414	173.2828	173.9495	178.3495	175.1148	0.1106	0.0000
	QXLD	$\hat{\phi} = 832.204$ $\hat{\gamma} = 0.2495$	95.5766	195.1532	195.4775	196.3745	198.5309	0.7091	0.0006
	PXLD	$\hat{\phi} = 9.9999$ $\hat{\gamma} = 0.0998$	429.1026	862.2051	862.5294	865.5829	863.4264	0.4881	0.0001
	IXLD	$\hat{\phi} = 0.7605$	0.7605	661.3023	661.3856	662.0304	663.2143	0.6606	0.0000

Table 5 shows goodness of fit tested with the four retrieved datasets. the first dataset shows that NGXLD performed better with the lowest AIC (796.1941), kolmogorov smirnov value 0.1347 indicate that the empirical and theoretical distributions are closer, suggesting a better fit. The KS p-value (0.0000), shows that the difference observed is statistical significant suggesting it is the best model for this dataset among the model consider. Other models like QXLD, PXLD and IXLD have higher AIC and negative log-likelihood, making them less preferred. The second and the other datasets performed the same, as NGXLD shows lowest value of *-ll*, *AIC*, *AICC*, *HQIC*, *BIC*, *K-S*.

IV. CONCLUSION

This paper aim to introduce New Generalized XLindley Distribution (*NGXLD*) by incorporation of weighted mechanism δ into Quasi Xlindley Distribution. The parameter is the weighted version of X called the size-biased version of X. The mathematical properties such as the method of moment, the coefficient of variation, the survival function, the hazard function were discussed and Maximum likelihood estimate was used to estimate the parameters. Finally, four datasets were used to check real life applicability of the NGXLD. The negative log-likelihood, criterion values such as *AIC*, *AICC*, *HQIC* and *BIC* were used for comparison with *QXL*, *ATPLD*, *WQLD*, and *MQLD*. The results from table 5 revealed that *NGXLD* produced low values in terms of *-LL*, *AIC*, *AICC*, *HQIC*, *BIC*, *K-S* and with significant value of *KS P-value* which shows that comparatively *NGXLD* is more efficient than the existing models considered. Hence, the *NGXLD* can be considered an important life time distribution for modelling life time data.

V. FUTURE RESEARCH

Having consider NGXLD as an important lifetime distribution for modeling data, it is therefore suggested that future research should include bias-correction techniques such as Analytical bias correction, Boostraps bias correction, simulation-based correction to accommodate small sample size and/or alternative estimation techniques such bayesian estimation to incorporate prior knowledge, penalized likelihood estimation to balance bias and variance.

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